

SOME GENERALIZATIONS OF MARONI'S INEQUALITY ON TIME SCALES

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Abstract. The familiar Opial's inequality is of great interest in differential and difference equations, and is also potentially useful in other areas of mathematical sciences. In an earlier work, we systematically investigated several general Opial-type inequalities on time scales. Here, in this sequel, we propose to present some extensions and generalizations of Maroni's inequality to hold true on time.

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