

ON THE EXISTENCE OF LINEAR AND BILINEAR MULTIPLIERS ON LORENTZ SPACES

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Abstract. First we show that any translation invariant bounded linear operator from $L^{p,t}(\mathbf{R})$ the Lorentz space on \mathbf{R} to $L^{p,s}(\mathbf{R})$ ($1 < p < \infty$, $1 \leq s < t < \infty$) is trivial, whose result improves Blozinski's result [4]. Next let ϕ be a bounded continuous function on \mathbf{R}^2 , and

$$T_\phi(f, g)(x) = \int \int \phi(\xi, \eta) \hat{f}(\xi) \hat{g}(\eta) e^{ix(\xi+\eta)} d\xi d\eta$$

the bilinear operator on Lorentz spaces. Then, we prove that the bounded bilinear operator T_ϕ is trivial in some cases of Lorentz spaces.

Mathematics subject classification (2010): Primary 43A22; secondary 42A45.

Keywords and phrases: Lorentz spaces, bilinear operator.

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