

## GENERALIZATIONS OF FATOU'S AND MARCINKIEWICZ'S RESULTS

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*Abstract.* Two new criteria for almost everywhere, uniform, and  $L^1$ -convergence of Fourier series are established. The first one is a Tauberian result for  $(C, 1)$  summability, which generalizes a result of Fatou. The second one is a norm analogue of Dini's test, which generalizes a result of Marcinkiewicz. It is also pointed out that our second result is not comparable with the Dini-Lipschitz test.

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*Keywords and phrases:* Fourier series; almost everywhere convergence;  $L^1$ -convergence; uniform convergence; the Dini test; the Dini-Lipschitz test; Fatou's theorem; Marcinkiewicz's theorem.

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