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BERNSTEIN–DOETSCH TYPE RESULTS FOR h–CONVEX FUNCTIONS

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Abstract. In this paper we introduce a class of h-convex functions which is a common generalization of the convexity, s-convexity, the Godunova-Levin functions and the P-functions. Namely, an h-convex function is defined as a function $f : D \to \mathbb{R}$ (where D is an open, convex, nonempty subset of a linear space) which satisfies

$$f(\lambda x + (1-\lambda)y) \leqslant h(\lambda) f(x) + h(1-\lambda) f(y),$$

for all $\lambda \in [0,1]$ and $x, y \in D$, where h is a given real function.

In this paper some regularity and Bernstein-Doetsch type results for h-convex functions are presented.

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