Mathematical
Inequalities
& Applications
Volume 14, Number 3 (2011), 509–522

THE MODULAR INTERPOLATION INEQUALITY IN SOBOLEV SPACES WITH VARIABLE EXPONENT ATTAINING THE VALUE 1

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Abstract. We prove a modular type interpolation inequality for functions belonging to Sobolev spaces with variable exponent attaining the value 1. The approach combines the original proof of the interpolation inequality by Nirenberg [19] with an inequality for averages over balls, avoiding the use of the norm interpolation inequality for variable exponent Sobolev spaces, known for exponents whose infimum is greater than 1.

Mathematics subject classification (2010): 26D10, 46E30, 46E35.
Keywords and phrases: Variable Sobolev space, variable Lebesgue space, interpolation inequalities, modular inequalities.

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