SAITOH'S INEQUALITY AND OPIAL'S INEQUALITY

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Abstract. We prove an elementary integral inequality which extends a norm inequality of Saitoh concerning absolutely continuous functions on an interval of the real axis. Our inequality immediately yields well-known Opial-type inequalities.

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