

## ON THE WEIGHTED HARDY TYPE INEQUALITY IN A FIXED DOMAIN FOR FUNCTIONS VANISHING ON THE PART OF THE BOUNDARY

YU. O. KOROLEVA

*Abstract.* We derive and discuss a new two-dimensional weighted Hardy-type inequality in a rectangle for the class of functions from the Sobolev space  $H^1$  vanishing on small alternating pieces of the boundary.

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