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CHARACTERIZATIONS OF THE CONVERGENCE OF HARMONIC AVERAGES OF DOUBLE NUMERICAL SEQUENCES

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Abstract. In recent years, the almost sure central limit theorem has attracted widespread attention in Probability Theory. It involves the harmonic (also called logarithmic) averages of a certain numerical sequence formed from a sequence of independent, identically distributed random variables. The convergence behavior of the sequence of harmonic averages of a given numerical sequence was studied in [3] by the third author. Our main goal in this paper is to extend these characterization results from single to double numerical sequences of complex numbers.

Among others, the following Theorem 2^* is proved. Let $\{x_{ij} : i, j = 1, 2, \ldots\}$ be a double sequence of complex numbers. Necessary and sufficient condition for the existence of the bounded limit relation

$$\mathrm{b}-\lim_{k,\ell\to\infty} \frac{1}{(\ln k)(\ln \ell)} \sum_{i=1}^{k} \sum_{j=1}^{\ell} \frac{x_{ij}}{ij} = \xi$$

is that

$$\mathrm{b}-\lim_{m,n\to\infty} \frac{1}{2^{m+n}} \max_{k\in J_m, \ell\in J_n} \left| \sum_{i=\mu_{m-1}+1}^{k} \sum_{j=\mu_{n-1}+1}^{\ell} \frac{x_{ij}-\xi}{ij} \right| = 0,$$

where

$$J_m := \{\mu_{m-1}+1, \mu_{m-1}+2, \ldots, \mu_m\}, \quad \mu_m := 2^{2^m}, \quad m = 0, 1, \ldots .$$

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