CHARACTERIZATIONS OF THE CONVERGENCE OF HARMONIC AVERAGES OF DOUBLE NUMERICAL SEQUENCES

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Abstract. In recent years, the almost sure central limit theorem has attracted widespread attention in Probability Theory. It involves the harmonic (also called logarithmic) averages of a certain numerical sequence formed from a sequence of independent, identically distributed random variables. The convergence behavior of the sequence of harmonic averages of a given numerical sequence was studied in [3] by the third author. Our main goal in this paper is to extend these characterization results from single to double numerical sequences of complex numbers.

Among others, the following Theorem 2∗ is proved. Let \( \{x_{ij} : i, j = 1, 2, \ldots \} \) be a double sequence of complex numbers. Necessary and sufficient condition for the existence of the bounded limit relation

\[
\lim_{k, \ell \to \infty} \frac{1}{(\ln k)(\ln \ell)} \sum_{i=1}^{k} \sum_{j=1}^{\ell} x_{ij} = \xi
\]

is that

\[
\lim_{m, n \to \infty} \max_{k \in J_m, \ell \in J_n} \left| \sum_{i=\mu_{m-1}+1}^{\mu_m} \sum_{j=\mu_{n-1}+1}^{\mu_n} x_{ij} - \xi \right| = 0,
\]

where

\[
J_m := \{\mu_{m-1} + 1, \mu_{m-1} + 2, \ldots, \mu_m\}, \quad \mu_m := 2^{2m}, \quad m = 0, 1, \ldots.
\]

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REFERENCES