A MAXIMAL INEQUALITY FOR NONNEGATIVE
SUB- AND SUPERMARTINGALES

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Abstract. Let \( X = (X_t)_{t \geq 0} \) be a nonnegative semimartingale and \( H = (H_t)_{t \geq 0} \) be a predictable process taking values in \([-1,1]\). Let \( Y \) denote the stochastic integral of \( H \) with respect to \( X \). We show that

(i) If \( X \) is a supermartingale, then
\[
\left\| \sup_{t \geq 0} Y_t \right\|_1 \leq 3 \left\| \sup_{t \geq 0} X_t \right\|_1
\]
and the constant 3 is the best possible.

(ii) If \( X \) is a submartingale satisfying \( \left\| X \right\|_\infty \leq 1 \), then
\[
\left\| \sup_{t \geq 0} Y_t \right\|_p \leq 2 \Gamma(p+1)^{1/p}, \quad 1 \leq p < \infty.
\]
The constant \( 2 \Gamma(p+1)^{1/p} \) is the best possible.

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REFERENCES