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# A MAXIMAL INEQUALITY FOR NONNEGATIVE SUB– AND SUPERMARTINGALES

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*Abstract.* Let $X = (X_t)_{t\geqslant 0}$ be a nonnegative semimartingale and $H = (H_t)_{t\geqslant 0}$ be a predictable process taking values in $[-1,1]$. Let $Y$ denote the stochastic integral of $H$ with respect to $X$. We show that

(i) If $X$ is a supermartingale, then

$$||\sup_{t\geqslant 0} Y_t||_1 \leqslant 3||\sup_{t\geqslant 0} X_t||_1$$

and the constant $3$ is the best possible.

(ii) If $X$ is a submartingale satisfying $||X||_\infty \leqslant 1$, then

$$||\sup_{t\geqslant 0} Y_t||_p \leqslant 2\Gamma(p+1)^{1/p}, \qquad 1 \leqslant p < \infty.$$

The constant $2\Gamma(p+1)^{1/p}$ is the best possible.

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