

## ON SOME GRONWALL TYPE INEQUALITIES INVOLVING ITERATED INTEGRALS

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*Abstract.* In this paper we consider simple inequalities involving iterated integrals in the inequality (1.1) for functions when the function  $u$  in the both side of the inequality (1.1) are replaced by the function  $w(u)$  and  $\varphi(u)$  for some functions  $w, \varphi$  and provide some retarded integral inequalities involving iterated integrals. Some applications are also given to illustrate the usefulness of our results.

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