

## SOME COMPLETELY MONOTONIC FUNCTIONS RELATED TO THE PSI FUNCTION

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*Abstract.* Complete monotonicity properties of some functions involving the psi function are studied and some known results are extended and generalized. Moreover, a necessary and sufficient conditions for some functions to be completely monotonic are presented and proved.

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