OPTIMAL EMBEDDINGS OF GENERALIZED INHOMOGENEOUS SOBOLEV SPACES ON $\mathbf{R}^n$

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Abstract. We prove optimal embeddings in the subcritical case of the inhomogeneous Sobolev spaces built-up over function spaces in $\mathbf{R}^n$ with $K-$ monotone and rearrangement invariant norm into another rearrangement invariant function spaces. The investigation is based on pointwise and integral estimates of the rearrangement or the oscillation of the rearrangement of f in terms of the rearrangement of the derivatives of f.

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