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POLARIZATION OF AN INEQUALITY

IVO KLEMEŠ

Abstract. We generalize a previous inequality related to a sharp version of the Littlewood conjecture on the minimal L_1 -norm of N -term exponential sums f on the unit circle. The new result concerns replacing the expression $\log(1+t|f|^2)$ with $\log\left(\sum_{k=1}^{K} t_k|f_k|^2\right)$. The proof occurs on the level of finite Toeplitz matrices, where it reduces to an inequality between their polarized determinants (or "mixed discriminants").

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