A POWER MEAN INEQUALITY FOR THE GRÖTZSCH RING FUNCTION

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Abstract. The Grötzsch ring function has numerous applications in geometric function theory and its properties have been investigated by many authors. Here we extend an earlier functional inequality involving the Grötzsch ring function and the geometric mean, due to Anderson, Vamanamurthy and Vuorinen, to the case of power mean.

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