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SINGULAR INTEGRODIFFERENTIAL INEQUALITIES AND APPLICATION TO FRACTIONAL DIFFERENTIAL PROBLEMS

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Abstract. In this paper we are concerned with an integrodifferential problem which arises for instance when we study a Cauchy-type fractional differential equation. This problem involves a convolution of a kernel with a nonlinear function of the solution together with its derivatives up to order two. For ordinary (third order) differential equations the kernel is regular while in our case it is singular and nonintegrable. Combining a desingularization technique due to the second author with some other estimations, we find bounds for solutions of the problem with different nonlinearities. Our results are illustrated by an application to fractional differential equations.

Mathematics subject classification (2010): 26D15, 26A51, 32F99, 41A17.
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