

ON SCHUR-CONVEXITY AND SCHUR-GEOMETRIC CONVEXITY OF FOUR-PARAMETER FAMILY OF MEANS

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Abstract. We prove that the four-parameter family of means

$$R(u, v; r, s; x, y) = \left[\frac{E(r, s; x^u, y^u)}{E(r, s; x^v, y^v)} \right]^{1/(u-v)}$$

is Schur-geometrically convex (concave) in x, y if $(u + v)(r + s) \geq (\leq) 0$, and Schur-concave (convex) in $u, v \geq 0$ if $r + s \geq (\leq) 0$.

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REFERENCES

- [1] P.S. BULLEN, *Handbook of Means and their Inequalities*, Kluwer Academic Press, Dordrecht, 2003.
- [2] Y. CHU, X. ZHANG AND G. WANG, *The Schur Geometrical Convexity of the Extended Mean Values*, *Journal of Convex Analysis* **15** (2008), 707–718.
- [3] A.W. MARSHALL AND I. OLKIN, *Inequalities: Theory of Majorization and Its Applications*, Academic Press, New York, 1979.
- [4] F. QI, *A note on Schur-convexity of extended mean values*, *Rocky Mountain J. Math.* **35**, 5 (2005), 1787–1793.
- [5] J. SÁNDOR, *The Schur-convexity of Stolarsky and Gini means*, *Banach J. Math. Anal.* **1**, 2 (2007), 212–215.
- [6] H.-N. SHI, Y.-M. JIANG AND W.-D. JIANG, *Schur-convexity and Schur-geometrically concavity of Gini means*, *Comput. Math. Appl.* **57**, 2 (2009), 266–274.
- [7] K.B. STOLARSKY, *Generalizations of the logarithmic mean*, *Math. Mag.* **48** (1975), 87–92.
- [8] A. WITKOWSKI, *Comparison theorem for two-parameter means*, *Math. Inequal. Appl.* **12**, 1 (2009), 11–20.
- [9] A. WITKOWSKI, *Monotonicity and convexity of S-means*, *Math. Inequal. Appl.* **13**, 1 (2010), 33–42.