

GENERAL INEQUALITIES FOR MULTIPOINT PADÉ APPROXIMANTS TO A STIELTJES FUNCTION EXPANDED AT REAL POINTS

S. TOKARZEWSKI AND E. WAJNRYB

Abstract. In this paper we establish the general inequalities for diagonal and subdiagonal multipoint Padé approximants to a Stieltjes function f in terms of power expansion of f on the real line. The inequalities derived produce the best upper and lower bounds on f with respect to the given coefficients of Stieltjes series. As an example of applications sequences of upper and lower Padé bounds converging to the effective dielectric constant of a random array of spheres are evaluated.

Mathematics subject classification (2010): 11J70, 41A21.

Keywords and phrases: N -point Padé approximants, Stieltjes functions, continued fractions.

REFERENCES

- [1] G. BAKER, JR, *Essentials of Padé Approximants*, Academic Press, New York, USA, 1975.
- [2] G. A. BAKER, JR AND P. GRAVES-MORRIS, *Padé Approximants*, volume 59 of Encyclopedia of Mathematics and its Applications, Cambridge University Press, London, second edition, 1996.
- [3] M. BARNESLEY, *The bounding properties of the multipoint Padé approximant to a series of Stieltjes on the real line*, J. Math. Phys., **14**, 3 (1973).
- [4] A. BULTHEEL, P. GONZÁLES-VERA, E. HENDRICKSEN, AND O. NJASTAD, *Monotonicity of multipoint padé approximants*, Preprint of paper presented at ICRA99, pages 1–12, 1999. <http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.43.7800>.
- [5] A. BULTHEEL, P. GONZÁLES-VERA, E. HENDRIKSEN, AND O. NJASTAD, *Orthogonal rational functions on the real half line with poles in $[-\infty, 0]$* . J. Comput. Appl. Math., **179** (2005), 121–151.
- [6] A. BULTHEEL, P. GONZÁLES-VERA, AND R. ORIVE, *Quadrature on the half-line and two-point Padé approximants to Stieltjes functions, Part I, Algebraic aspects*, J. of Comp. Appl. Math., **65** (1995), 57–72.
- [7] B. CICHOCKI AND B. U. FELDERHOF, *Electrostatic spectrum and dielectric constant of nonpolar hard sphere fluids*, J. Chem. Phys., **90**, 9 (May 1989), 4960–496.
- [8] B. CICHOCKI AND B. U. FELDERHOF, *Cavity field and reaction in nonpolar fluids*, J. Chem. Phys., **92**, 10 (May 1992), 6104–6111.
- [9] J. GILEWICZ AND A. P. MAGNUS, *Optimal inequalities of Padé approximants errors in the Stieltjes case: Closing result*, Integral Transform. Spec. Funct., **1** (1993), 9–18.
- [10] K. GOLDEN AND G. PAPANICOLAOU, *Bounds for effective parameters of heterogeneous media by analytic continuation*, Comm. Math. Phys., **90**, 4 (1983), 473–491.
- [11] K. HINSEN AND B. FELDERHOF, *Dielectric constant of a suspension of uniform spheres*, Physical Review B, **46**, 20 (November 1992), 12955–12963.
- [12] S. TOKARZEWSKI, *N -point Padé approximants to real valued Stieltjes series with nonzero radii of convergence*, J. Comp. Appl. Math., **75** (1996), 259–280.
- [13] S. TOKARZEWSKI, *Two-point Padé approximants for the expansion of Stieltjes function in a real domain*, J. Comp. Appl. Math., **67** (1996), 59–72.
- [14] S. TOKARZEWSKI, A. MAGNUS, AND J. GILEWICZ, *Estimation of a Stieltjes function expanded to Taylor series at complex conjugate points*, J. Comp. Appl. Math., **233**, 3 (2009), 835–841.

- [15] S. TOKARZEWSKI AND J. J. TELEGA, *Bounds on effective moduli by analytical continuation of the Stieltjes function expanded at zero and infinity*, *Z. angew. Math. Phys.*, **48** (1997), 1–20.