

A NOTE ON THE WEIGHTED HARMONIC–GEOMETRIC–ARITHMETIC MEANS INEQUALITIES

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Abstract. In this note, we derive non trivial sharp bounds related to the weighted harmonic-geometric-arithmetic means inequalities, when two out of the three terms are known. As application, we give an explicit bound for the trace of the inverse of a symmetric positive definite matrix and an inequality related to the coefficients of polynomials with positive roots.

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