# TWO–PARAMETER SUNOUCHI OPERATOR WITH RESPECT TO CHARACTER SYSTEM OF $p$–SERIES FIELD IN THE KACZMARZ REARRANGEMENT

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*Abstract.* Let $G_p$ be $p$-series field. We prove the restricted two-parameter Sunouchi operator $T_h^\chi$ is bounded from $H_q^\square$ to $L_q$ for $0 < q \leqslant 1$. By means of interpolation and duality argument, this theorem can be extended to Hardy-Lorentz spaces. As a consequence, we prove the restricted Sunouchi operator is of weak type $(L_1, L_1)$.

*Mathematics subject classification* (2010): 42C10.
*Keywords and phrases*: Character system, Kaczmarz rearrangement, Sunouchi operator.

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