

VORONOVSKAJA'S THEOREM FOR SCHOENBERG OPERATOR

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Abstract. In this paper we represent new quantitative variants of Voronovskaja's Theorem for Schoenberg variation-diminishing spline operator. We estimate the rate of uniform convergence for $f \in C^2[0, 1]$ and generalize the results obtained earlier by Goodman, Lee, Sharma, Gonska etc.

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