BOUNDS FOR LINEAR FUNCTIONALS ON
MONOTONE FUNCTIONS IN L^p–SPACES

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Abstract. We consider the $L^p[a,b]$ space of functions which are integrable in the p-th power on a finite interval $[a,b]$, for $1 \leqslant p < \infty$. We establish optimal bounds on continuous linear functionals over this space, imposing the restrictions on elements of the space, which are assumed to be nondecreasing, integrable to zero, with the unit norm. We mention some applications of the bounds in the probability and statistics.

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