

ESTIMATE FOR THE DISCRETE TIME HEDGING ERROR OF THE AMERICAN OPTION ON A DIVIDEND-PAYING STOCK

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Abstract. This work is devoted to the discrete time hedging of the American option on a dividend-paying stock with a convex payoff, the particular case of which is American call option. Perfect hedging requires continuous trading in time and knowledge of the partial derivative of the value function of the American option in the underlying asset. Neither one can trade continuously in time nor the closed-form expression of the value function of the American option is known.

Several approximation methods have been developed for the calculation of this unknown value function. We justify in this work that having at hand any such nonnegative uniform approximation, it is possible to construct a discrete time hedging strategy the value process of which uniformly approximates the value process of the corresponding continuous time perfect hedging portfolio.

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