

## CONCENTRATION–COMPACTNESS PRINCIPLE FOR EMBEDDING INTO MULTIPLE EXPONENTIAL SPACES

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*Abstract.* Let  $\Omega \subset \mathbb{R}^n$ ,  $n \geq 2$ , be a bounded domain and let  $\alpha < n - 1$ . We prove the Concentration-Compactness Principle for the embedding of the Orlicz-Sobolev space  $W_0^1 L^n \log^{n-1} L \log^\alpha \log L(\Omega)$  into the Orlicz space corresponding to a Young function that behaves like  $\exp(\exp(t^{\frac{n}{n-1-\alpha}}))$  for large  $t$ . We also give the result for the case of the embedding into triple and other multiple exponential spaces.

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