

TWO FAMILIES OF CYCLIC INEQUALITIES

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Abstract. In this paper two families of cyclic inequalities in three variables are studied. More precisely, necessary and sufficient conditions in order that the cyclic sums $\sum x^m(x+y)^n$ and $\sum s_\alpha(x)s_\beta(x+y)$ are non-negative are stated and proven. Here m, n are positive integers, α, β are positive real numbers and $s_\alpha(x) = x|x|^{\alpha-1}$, $x \in \mathbb{R}^*$, $s_\alpha(0) = 0$.

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