

HÖLDER MEAN INEQUALITIES FOR THE GENERALIZED GRÖTZSCH RING AND HERSCH–PFLUGER DISTORTION FUNCTIONS

SONG-LIANG QIU, YE-FANG QIU, MIAO-KUN WANG AND YU-MING CHU

Abstract. In this paper, we study the monotonicity properties of the generalized elliptic integrals and establish two Hölder mean inequalities for the generalized Grötzsch ring function $\mu_a(r)$ and the generalized Hersch-Pfluger distortion function $\varphi_K^a(r)$.

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