

INEQUALITIES FOR THE NORMS OF FINITE DIFFERENCE OPERATORS OF MULTIPLY MONOTONE SEQUENCES

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Abstract. In this paper we shall present discrete Kolmogorov type inequalities for multiply monotone sequences defined on non-positive integers. Moreover, we will provide a more delicate information by obtaining the description of the following modulus of continuity

$$\omega_{p,q}^{k,j,r}(\delta, \varepsilon) = \sup\{\|\Delta^k x\|_q : x, \Delta x, \dots, \Delta^j x \geq 0, \|x\|_p = \delta, \|\Delta^r x\|_\infty = \varepsilon\}$$

for $\delta \geq \varepsilon > 0$ and values of $j = r - 2$ or $j = r - 1$ depending on values of other parameters.

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REFERENCES

- [1] V. F. BABENKO, YU. V. BABENKO, *Kolmogorov inequalities for multiply monotone functions defined on a half-line*, East J. Approx. **11**, 2 (2005), 169–186.
- [2] V. F. BABENKO, N. P. KORNEICHUK, V. A. KOFANOV, S. A. PICHUGOV, *Inequalities for derivatives and their applications*, Naukova Dumka, Kiev, 2003 (Russian).
- [3] Z. DITZIAN, *Discrete and shift Kolmogorov type inequalities*, Proc. Roy. Soc. Edinburgh, Sect. A **93**, 3–4 (1982/83), 307–317.
- [4] H. G. KAPER, B. E. SPELLMAN, *Best constants in norm inequalities for the difference operator*, Trans. Amer. Math. Soc. **299**, 1 (1987), 351–372.
- [5] A. N. KOLMOGOROV, *On inequalities between upper bounds of consecutive derivatives of arbitrary function on the infinite interval*, Scientific Notes MGU **30** (1939), 3–16 (in Russian).
- [6] A. KOLMOGOROFF, *Une generalisation de J.Hadamard entre les bornes superier des derivees secce-sives d'une fonction*, C. R. Acad. Sci. **207** (1938), 764–765.
- [7] N. P. KORNEICHUK, V. F. BABENKO, A. A. LIGUN, *Extremal properties of polynomials and splines*, Kiev, Naukova Dumka, 1992.
- [8] M. K. KWONG, A. ZETTL, *Ramifications of Landau's inequality*, Proc. Roy. Soc. Edinburgh **86** (1980), 175–212.
- [9] M. K. KWONG, A. ZETTL, *Landau's inequality for the difference operator*, Proc. Amer. Math. Soc. **104**, 1 (1988), 201–206.
- [10] M. K. KWONG, A. ZETTL, *Norm inequalities for derivatives and differences*, Lecture Notes in Mathematics, 1536. Springer-Verlag, Berlin, 1992. viii+150 pp.
- [11] A. W. MARSHALL, I. OLKIN, *Inequalities: theory of majorization and its applications*, Mathematics in Science and Engineering, Vol. 143. Academic Press, 1979.
- [12] V. M. OLOVYANISHNIKOV, *To the question on inequalities between upper bounds of consecutive derivatives on a half-line*, Uspekhi Mat. Nauk **6**, 2(42) (1951), 167–170.
- [13] J. VELIKINA, *Sharp constants in discrete Kolmogorov type inequalities*, East J. Approx. **5**, 1 (1999), 89–100.
- [14] J. V. VELIKINA, *Some sharp inequalities and estimates of approximation of functions of discrete argument by splines*, Ph. D. thesis, Dnepropetrovsk National University, Ukraine, 1999.