

## ON WILKER AND HUYGENS TYPE INEQUALITIES

EDWARD NEUMAN

*Abstract.* Trigonometric inequalities, which have been obtained by J. Wilker [11] and C. Huygens [4], have attracted attention of several researchers (see, e.g., [1], [3], [6], [7], [8], [9], [10], [12], [13], [14], [15], [16]). In this paper we offer several refinements and generalizations of these results.

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