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ON STRONG $(\alpha,\mathbb{F})$–CONVEXITY

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Abstract. In this paper, strongly (α,T)-convex functions, i.e., functions $f : D \to \mathbb{R}$ satisfying the functional inequality

$$f(tx+(1-t)y) \leqslant tf(x)+(1-t)f(y)-t\alpha\big((1-t)(x-y)\big)-(1-t)\alpha\big(t(y-x)\big)$$

for $x,y \in D$ and $t \in T\cap[0,1]$ are investigated. Here D is a convex set in a linear space, α is a nonnegative function on $D-D$, and $T \subseteq \mathbb{R}$ is a nonempty set. The main results provide various characterizations of strong (α,T)-convexity in the case when T is a subfield of $\mathbb{R}$.

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REFERENCES

[1] F. BERNSTEIN AND G. DOETSCH, *Zur Theorie der konvexen Funktionen*, Math. Ann. **76**, no. 4 (1915), 514–526. MR 1511840
[2] Z. BOROS AND ZS. PÁLES, $\mathbb{Q}$*-subdifferential of Jensen-convex functions*, J. Math. Anal. Appl. **321**, no. 1 (2006), 99–113. MR 2007c:49017
[3] R. GER, *Almost approximately convex functions*, Math. Slovaca **38**, no. 1 (1988), 61–78. MR 89m:26020a
[4] R. B. HOLMES, *Geometric Functional Analysis and its Applications*, Graduate Texts in Mathematics, vol. 24, Springer-Verlag, Berlin–Heidelberg–New York, 1975. MR 53 #14085
[5] M. KUCZMA, *An Introduction to the Theory of Functional Equations and Inequalities*, Prace Naukowe Uniwersytetu Śląskiego w Katowicach, vol. 489, Państwowe Wydawnictwo Naukowe — Uniwersytet Śląski, Warszawa–Kraków–Katowice, 1985. MR 86i:39008
[6] J. MAKÓ AND ZS. PÁLES, *Strengthening of strong and approximate convexity*, Acta Math. Hungar., accepted (2011).
[7] S. MAZUR AND W. ORLICZ, *Sur les espaces métriques linéaires. II*, Studia Math. **13** (1953), 137–179. MR 16,932e
[8] N. MERENTES AND K. NIKODEM, *Some remarks on strongly convex functions*, Aequationes Math. **80** (2010), 193–199
[9] G. J. MINTY, *On the monotonicity of the gradient of a convex function*, Pacific J. Math. **14** (1964), 243–247. MR 0167859 (29 #5125a)
[10] J. MROWIEC, JA. TABOR, AND JÓ. TABOR, *Approximately midconvex functions*, Inequalities and Applications (Noszvaj, 2007) (C. Bandle, A. Gilányi, L. Losonczi, M. Plum, and Zs. Páles, eds.), International Series of Numerical Mathematics, vol. 157, Birkhäuser Verlag, 2008, pp. 261–267.
[11] A. MUREŃKO, JA. TABOR, AND JÓ. TABOR, *Applications of de Rham Theorem in approximate midconvexity*, J. Diff. Equat. Appl. (2011), accepted.
[12] K. NIKODEM, ZS. PÁLES, AND SZ. WĄSOWICZ, *Abstract separation theorems of Rodé type and their applications*, Ann. Polon. Math. **72**, no. 3 (1999), 207–217. MR 2001c:26013
[13] E. S. POLOVINKIN, *Strongly convex analysis*, Mat. Sb. **187**, no. 2 (1996), 103–130. MR 97k:52004
[14] B. T. POLYAK, *Existence theorems and convergence of minimizing sequences for extremal problems with constraints*, Dokl. Akad. Nauk SSSR **166** (1966), 287–290. MR 33 #6466
[15] A. W. ROBERTS AND D. E. VARBERG, *Convex Functions*, Pure and Applied Mathematics, vol. 57, Academic Press, New York–London, 1973. MR 56 #1201

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[16] R. T. ROCKAFELLAR, *Characterization of the subdifferentials of convex functions*, Pacific J. Math. **17** (1966), 497–510. MR 33 #1769
[17] R. T. ROCKAFELLAR, *Convex Analysis*, Princeton Mathematical Series, No. 28, Princeton University Press, Princeton, N.J., 1970. MR 43 #445
[18] R. T. ROCKAFELLAR, *On the maximal monotonicity of subdifferential mappings*, Pacific J. Math. **33** (1970), 209–216. MR 41 #7432
[19] G. RODÉ, *Eine abstrakte Version des Satzes von Hahn–Banach*, Arch. Math. (Basel) **31** (1978), 474–481.
[20] JA. TABOR AND JÓ. TABOR, *Generalized approximate midconvexity*, Control Cybernet. **38**, no. 3 (2009), 655–669.
[21] JA. TABOR, JÓ. TABOR, AND M. ŻOŁDAK, *Approximately convex functions on topological vector spaces*, Publ. Math. Debrecen **77**, no. 1-2 (2010), 115–123. MR 2675738
[22] J.-PH. VIAL, *Strong convexity of sets and functions*, J. Math. Econom. **9** (1982), 187–205. MR 82m:52004