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APPROXIMATION BY NÖRLUND MEANS OF DOUBLE WALSH–FOURIER SERIES FOR LIPSCHITZ FUNCTIONS

KÁROLY NAGY

Abstract. For double trigonometric Fourier series, Móricz and Rhoades studied the rate of uniform approximation by Nörlund means of the rectangular partial sums of double Fourier series of a function belonging to the class Lipα $(0 < \alpha \leqslant 1)$ [12] and the class of continuous functions [13], on the two-dimensional torus. As a special case, they obtained the rate of uniform approximation by double Cesàro means.

The main aim of this paper is to investigate the rate of the approximation by the Nörlund means $T^w_{m,n}(f)$ of double Walsh-Fourier series of a function in L^p, in particular, in $\mathrm{Lip}(\alpha, p)$, where $\alpha > 0$ and $1 \leqslant p \leqslant \infty$. In case $p = \infty$, by L^p we mean C_W, the collection of the uniform W-continuous functions.

Earlier results on one-dimensional Nörlund means of the Walsh-Fourier series was given by Móricz and Siddiqi [16].

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