

## SCORES, INEQUALITIES AND REGULAR HYPERTOURNAMENTS

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*Abstract.* A  $k$ -hypertournament is a complete  $k$ -hypergraph with each  $k$ -edge endowed with an orientation, that is, a linear arrangement of the vertices contained in the edge. In a  $k$ -hypertournament, the score  $s_i$  (losing score  $r_i$ ) of a vertex  $v_i$  is the number of edges containing  $v_i$  in which  $v_i$  is not the last element (in which  $v_i$  is the last element). In this paper we obtain inequalities involving powers of scores and losing scores by using classical results from mathematical analysis (such as Hölder's inequality) and show that equality holds if and only if the hypertournament is regular. We then use these inequalities to give a short proof of a result on the existence of regular hypertournaments. We also obtain an upper bound on the number of directed paths of length 2 in tournaments and hypertournaments, prove that the bound is sharp and that it is realized by regular hypertournaments.

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