A GEOMETRIC PROOF OF BLUNDON'S INEQUALITIES

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Abstract. A geometric approach of Blundon's inequality is presented. Theorem 2.1 gives the formula for $\cos\widehat{ION}$ in terms of the symmetric invariants s, R, r of a triangle, implying Blundon's inequality (Theorem 2.2). A dual formula for $\cos\widehat{I_aON_a}$ is given in Theorem 3.1 and this implies the dual Blundon's inequality (Theorem 3.2). As applications, some inequalities involving the exradii of the triangle are presented in the last section.

Mathematics subject classification (2010): 26D05; 26D15; 51N35..

Keywords and phrases: Euler's inequality; fundamental triangle inequality; Euler determination problem; dual fundamental triangle inequality.

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