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NEUMANN EIGENVALUE SUMS ON TRIANGLES ARE (MOSTLY) MINIMAL FOR EQUILATERALS

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Abstract. We prove that among all triangles of given diameter, the equilateral triangle minimizes the sum of the first n eigenvalues of the Neumann Laplacian, when $n \geqslant 3$.

The result fails for $n = 2$, because the second eigenvalue is known to be minimal for the degenerate acute isosceles triangle (rather than for the equilateral) while the first eigenvalue is 0 for every triangle. We show the third eigenvalue is minimal for the equilateral triangle.

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