BEST POSSIBLE INEQUALITIES AMONG HARMONIC, GEOMETRIC, LOGARITHMIC AND SEIFFERT MEANS

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Abstract. In this paper, we find the greatest value α and the least values β, p, q and r in $(0,1/2)$ such that the inequalities $L(\alpha a + (1-\alpha)b, \alpha b + (1-\alpha)a) < P(a,b) < L(\beta a + (1-\beta)b, \beta b + (1-\beta)a)$, $H(pa + (1-p)b, pb + (1-p)a) > G(a,b)$, $H(qa + (1-q)b, qb + (1-q)a) > L(a,b)$, and $G(ra + (1-r)b, rb + (1-r)a) > L(a,b)$ hold for all $a,b > 0$ with $a \neq b$. Here, $H(a,b)$, $G(a,b)$, $L(a,b)$ and $P(a,b)$ denote the harmonic, geometric, logarithmic and Seiffert means of two positive numbers a and b, respectively.

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REFERENCES

[1] H. ALZER AND S.-L. QIU, *Inequalities for means in two variables*, Arch. Math. **80** (2003), 201–215.
[2] F. BURK, *The geometric, logarithmic, and arithmetic mean inequality*, Amer. Math. Monthly **94** (1987), 527–528.
[3] B. C. CARLSON, *The logarithmic mean*, Amer. Math. Monthly **79** (1972), 615–618.
[4] Y.-M. CHU AND B.-Y. LONG, *Sharp inequalities between means*, Math. Inequal. Appl. **14** (2011), 647–655.
[5] Y.-M. CHU, Y.-F. QIU, M.-K. WANG AND G.-D. WANG, *The optimal convex combination bounds of arithmetic and harmonic means for the Seiffert's mean*, J. Inequal. Appl. **2010**, Article ID 436457, 7 pages.
[6] Y.-M. CHU AND W.-F. XIA, *Two optimal double inequalities between power mean and logarithmic mean*, Comput. Math. Appl. **60** (2010), 83–89.
[7] P. A. HÄSTÖ, *Optimal inequalities between Seiffert's mean and power means*, Math. Inequal. Appl. **7** (2004), 47–53.
[8] A. A. JAGERS, *Solution of Problem 887*, Nieuw Arch. Wisk. (4) **12** (1994), 230–231.
[9] E. B. LEACH AND M. C. SHOLANDER, *Extended mean values II*, J. Math. Anal. Appl. **92** (1983), 207–223.
[10] T. P. LIN, *The power mean and the logarithmic mean*, Amer. Math. Monthly **81** (1974), 879–883.
[11] H. LIU AND X.-J. MENG, *The optimal convex combination bounds for Seiffert's mean*, J. Inequal. Appl. **2011**, Article ID 686834, 9 pages.
[12] A. O. PITTENGER, *Inequalities between arithmetic and logarithmic means*, Univ. Beograd. Publ. Elektrotehn. Fak. Ser. Mat. Fiz. **678-715** (1980), 15–18.
[13] A. O. PITTENGER, *The symmetric, logarithmic and power means*, Univ. Beograd. Publ. Elektrotehn. Fak. Ser. Mat. Fiz. **678-715** (1980), 19–23.
[14] J. SÁNDOR, *On certain inequalities for means III*, Arch. Math. **76** (2001), 34–40.
[15] H.-J. SEIFFERT, *Problem 887*, Nieuw Arch. Wisk. (4) **11** (1993), 176–176.
[16] H.-J. SEIFFERT, *Ungleichungen für einen bestimmten Mittelwert*, Nieuw Arch. Wisk. (4) **13** (1995), 195–198.
[17] M.-K. WANG, Y.-F. QIU AND Y.-M. CHU, *Sharp bounds for Seiffert means in terms of Lehmer means*, J. Math. Inequal. **4** (2010), 581–586.
[18] S.-S. WANG AND Y.-M. CHU, *The best bounds of the combination of arithmetic and harmonic means for the Seiffert's mean*, Int. J. Math. Anal. **4** (2010), 1079–1084.

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