BEST POSSIBLE INEQUALITIES AMONG HARMONIC, GEOMETRIC, LOGARITHMIC AND SEIFFERT MEANS

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Abstract. In this paper, we find the greatest value $\alpha$ and the least values $\beta$, $p$, $q$ and $r$ in $(0,1/2)$ such that the inequalities $L(a\alpha + (1-\alpha)b, \alpha b + (1-\alpha)a) < P(a,b) < L(\beta a + (1-\beta)b, \beta b + (1-\beta)a)$, $H(pa + (1-p)b, pb + (1-p)a) > G(a,b)$, $H(qa + (1-q)b, qb + (1-q)a) > L(a,b)$, and $G(ra + (1-r)b, rb + (1-r)a) > L(a,b)$ hold for all $a,b > 0$ with $a \neq b$. Here, $H(a,b)$, $G(a,b)$, $L(a,b)$ and $P(a,b)$ denote the harmonic, geometric, logarithmic and Seiffert means of two positive numbers $a$ and $b$, respectively.


Keywords and phrases: harmonic mean, geometric mean, logarithmic mean, Seiffert mean.

REFERENCES