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OPPENHEIM'S INEQUALITY AND RKHS

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Abstract. Applying norm inequalities for RKHSs corresponding to the product of reproducing kernels and using the minimal norm of the Nevanlinna interpolation, we give the basic background and essences of the quite famous fundamental inequalities, Oppenheim's inequality, Hadamard's inequality and Schur's inequality on positive semidefinite matrices. In particular, as an application, we determine equality conditions for Oppenheim's inequality and Schur's inequality.

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