Mathematical
Inequalities
& Applications
Volume 15, Number 2 (2012), 469–486

SOME NEW GENERALIZATIONS OF ZYGMUND–TYPE INEQUALITIES FOR POLYNOMIALS

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Abstract. In this paper, we consider a problem of investigating the dependence of

$$\left\| P(Rz) - \alpha P(rz) + \beta \left\{ \left(\frac{R+1}{r+1} \right)^n - |\alpha| \right\} \right\|_p$$

on $\|P(z)\|_p$ for arbitrary real or complex numbers α, β with $|\alpha| \leqslant 1$, $|\beta| \leqslant 1$, $R > r \geqslant 1$, $p > 0$ and present certain sharp compact generalizations of some well-known Zygmund-type inequalities for polynomials, from which a variety of interesting results follows as special cases.

Mathematics subject classification (2010): 30D15, 41A17.
Keywords and phrases: polynomials, inequalities in the complex domain Zygmund's inequality.

REFERENCES

[1] N. C. ANKENY AND T. J. RIVLIN, *On a theorm of S. Bernstein*, Pacific J. Math. **5** (1955), 849–852.
[2] V. V. ARESTOV, *On integral inequalities for trigonometric polynimials and their derivatives*, Izv. Akad. Nauk SSSR Ser. Mat. **45** (1981), 3–22 [in Russian]. English translation; Math. USSR-Izv. **18** (1982), 1–17.
[3] A. AZIZ, *A new proof and a generalization of a theorem of De Bruijn*, Proc. Amer Math. Soc. **106** (1989), 345–350.
[4] A. AZIZ AND N. A. RATHER, *L^p inequalities for polynomials*, Glasnik Matematicki **32** (1997), 39–43.
[5] A. AZIZ AND N. A. RATHER, *On an inequality of S. Bernstein and Gauss-Lucas theorem, Analytic and Geometric inequalities*, Kluwer Acad. Pub., 1999, 29–35.
[6] A. AZIZ AND N. A. RATHER, *Some compact generalization of Zygmund-type inequalities for polynomials*, Nonlinear studies **6** (1999), 241–255.
[7] R. P. BOAS, JR., AND Q. I. RAHMAN, *L^p inequalities for polynomials and entire functions*, Arch. Rational Mech. Anal. **11** (1962), 34–39.
[8] N. G. BRUIJN, *Inequalities concerning polynomials in the complex domain*, Nederal. Akad. Wetensch. Proc. **50** (1947), 1265–1272.
[9] K. K. DEWAN AND N. K. GOVIL, *An inequality for self-inversive polynomials*, J. Math. Anal. Appl. **45** (1983), 490.
[10] G. H. HARDY, *The mean value of the modulus of an analytic function*, Proc. London Math. Soc. **14** (1915), 269–277.
[11] V. K. JAIN, *Generalization of certain well-known inequalities for polynomials*, Glasnik Matematicki **32** (1997), 45–51.
[12] P. D. LAX, *Proof of a conjecture of P.Erdös on the derivative of a polynomial*, Bull. Amer. Math. Soc. **50** (1944), 509–513.
[13] G. V. MILOVANOVIC, D. S. MITRINOVIC AND TH. M. RASSIAS, *Topics in Polynomials: Extremal Properties, Inequalities, Zeros*, World scientific Publishing Co., Singapore, 1994.
[14] G. PÓLYA AN G. SZEGÖ, *Aufgaben und lehrsätze aus der Analysis*, Springer-Verlag, Berlin, 1925.
[15] Q. I. RAHMAN AND G. SCHMEISSER, *Les Ineéqualitués de Markoff et de Bernstein*, Presses Univ. Montréal, Montréal, Quebec, 1983.

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[16] Q. I. RAHMAN AND G. SCHMESSIER, *L^p inequalities for polynomials*, J. Approx. Theory **53** (1988), 26–32.
[17] Q. I. RAHMAN AND G. SCHMESSIER, *Analytic theory of polynomials*, Claredon Press, Oxford, 2002.
[18] M. RIESZ, *Formula d'interpolation pour la dérivée d'un polynome trigonométrique*, C. R. Acad. Sci, Paris, **158** (1914), 1152–1254.
[19] A. C. SCHAFFER, *Inequalities of A. Markoff and S. Bernstein for polynomials and related functions*, Bull. Amer. Math Soc. **47** (1941), 565–579.
[20] A. ZYGMUND, *Aremark on conjugate series*, Proc. London Math. Soc. **34** (1932), 292–400.