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POINTWISE AND INTEGRAL ESTIMATES FOR THE FRACTIONAL INTEGRALS ON THE LAGUERRE HYPERGROUP

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Abstract. Let $\mathbb{K} = [0,\infty) \times \mathbb{R}$ be the Laguerre hypergroup which is the fundamental manifold of the radial function space for the Heisenberg group. In this paper, some pointwise and integral estimates for the fractional integrals in terms of the maximal and fractional maximal functions on the Laguerre hypergroup are obtained. Basing on these results, we prove interpolation theorems for the fractional maximal functions and fractional integrals, and the Sobolev theorem on the Laguerre hypergroup.

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