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SOLVING THE MATRIX INEQUALITY $AXB+(AXB)^* \geqslant C$

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Abstract. A pair of complex Hermitian matrices A and B of the same size are said to satisfy an inequality $A \geqslant B$ in the Löwner partial ordering if $A-B$ is nonnegative definite. In this note, we first derive the general solutions in closed-form for the linear matrix equation $AXB+(AXB)^*=C$ by using generalized inverses of matrices, and then derive general solutions of the linear matrix inequality $AXB+(AXB)^* \geqslant C$ when C is a Hermitian nonnegative definite matrix.

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