

## UNIVERSAL INEQUALITIES FOR EIGENVALUES OF QUADRATIC POLYNOMIAL OPERATOR OF THE KOHN LAPLACIAN

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*Abstract.* In this paper, we investigate the Dirichlet weighted eigenvalue problem of quadratic polynomial operator of the Kohn Laplacian on a bounded domain in the Heisenberg group  $\mathbb{H}^n$ . We establish two inequalities for eigenvalues of this problem. One of them implies an explicit estimate for the upper bound of the  $(k+1)$ -th eigenvalue in terms of the first  $k$  eigenvalues. Moreover, as a special case, we give some universal inequalities and estimates for eigenvalues of the bi-Kohn Laplacian.

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