Mathematical
Inequalities
& Applications
Volume 15, Number 3 (2012), 599–617
doi:10.7153/mia-15-53

A NEW GENERAL BOAS–TYPE INEQUALITY AND RELATED CAUCHY–TYPE MEANS

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Abstract. We prove a new Boas-type inequality in a context of topological spaces and general σ-finite Borel measures. This enables us to introduce an one-parameter class of non-negative Boas differences and examine their properties, such as continuity and log-convexity. By proving the related Galvani's theorem and mean-value theorems of the Lagrange and Cauchy type we establish a new class of two-parameter Cauch-type means.

Mathematics subject classification (2010): Primary 26D10; Secondary 26D15.
Keywords and phrases: Boas inequality, Hardy-Littlewood average, Boas differences, Cauchy means.

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