

## ON SOME CHARACTERIZATIONS OF THE PARTIAL ORDERINGS FOR BOUNDED OPERATORS

CHUNYUAN DENG AND SHUNQIN WANG

*Abstract.* In this paper, we investigate some common characterizations and various individual properties of the star ordering, the left star ordering, the right star ordering and the minus partial ordering of bounded operators on a Hilbert space. Some generalizations of results known in the literature and a number of new results for bounded operators are derived.

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