

ON PRODUCTS OF GENERALIZED ORLICZ SPACES

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Abstract. In the context of generalized Orlicz spaces, the products $X_{\Phi_1} \odot X_{\Phi_2}$ and $X_{\Phi_1} \otimes X_{\Phi_2}$ are studied and conditions are obtained under which these spaces are contained in a suitable space X_{Φ} . These imbedding results (inequalities) are in a sense sharp and for the case $X = L_1$, the conditions are even necessary and sufficient. Moreover, a new Hölder type inequality is proved.

Mathematics subject classification (2010): 26D10, 26D15, 46E35.

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REFERENCES

- [1] T. ANDO, *On product of Orlicz spaces*, Math. Annalen. **140** (1960), 174–186.
- [2] C. BENNETT AND R. SHARPLEY, *Interpolation of Operators*, Academic Press, London, 1988.
- [3] J. CERDA, H. HUDZIK AND M. MASTYLO, *On the geometry of some Calderón-Lozanovskii interpolation spaces*, Indag. Math. (N.S.) **6** (1995), 35–49.
- [4] P. FORALEWSKI AND H. HUDZIK, *Some basic properties of generalized Calderón-Lozanovskii spaces*, Collect. Math. **48** (1997), 523–538.
- [5] H. HUDZIK, A. KAMINSKA AND M. MASTYLO, *Geometric properties of some Calderón-Lozanovskii spaces and Orlicz-Lorentz spaces*, Houston J. Math. **22** (1996), 639–663.
- [6] P. JAIN, B. GUPTA AND D. VERMA, *Mean inequalities in certain Banach function spaces*, J. Math. Anal. Appl. **334**, 1 (2007), 358–367.
- [7] P. JAIN, L. E. PERSSON AND P. UPRETI, *Inequalities and properties of some generalized Orlicz classes and spaces*, Acta Math. Hungar. **117**, 1–2 (2007), 161–174.
- [8] P. JAIN AND P. UPRETI, *Certain properties of generalized Orlicz spaces*, J. Inequal Pure and Appl. Math. **10**, 2 (2009), 10 pp.
- [9] P. JAIN AND D. VERMA, *Two dimensional mean inequalities in certain Banach function spaces*, Real Anal. Exchange **33**, 1 (2008), 125–141.
- [10] A. KAMINSKA, L. MALIGRANDA AND L. E. PERSSON, *Indices and regularizations of measurable functions*, in: Function Spaces (Poznan, 1998), Lecture Notes in Pure and Appl. Math., 213, Dekker, New York, 2000, 231–246.
- [11] A. KAMINSKA, L. MALIGRANDA AND L. E. PERSSON, *Indices, convexity and concavity of Calderón-Lozanovskii spaces*, Math. Scand. **92** (2003), 141–160.
- [12] M. A. KRASNOSEL'SKII AND J. B. RUTICKII, *Convex Functions and Orlicz Spaces*, Noordhoff Ltd., Groningen, 1961.
- [13] A. KUFNER, J. OLDRIICH AND F. SVATOPLUK, *Function Spaces*, Noordhoff International Publishing, Leyden, 1977.
- [14] J. LINDENSTRAUSS AND L. TZAFRIRI, *Classical Banach Spaces II*, Springer-Verlag, 1979.
- [15] G. JA. LOZANOVSKII, *On reflexive spaces generalizing the reflexive space of Orlicz*, Dokl. Akad. Nauk SSSR **163** (1965), 573–576; English translation in: Soviet Math. Dokl. **6** (1965), 968–971.
- [16] G. JA. LOZANOVSKII, *Certain Banach lattices IV*, Sibirsk. Mat. Zh. **14** (1973), 140–155 (Russian); English translation in: Siberian Math. J. **14** (1973), 97–108.
- [17] W. A. J. LUXEMBURG, *Banach Function Spaces*, Ph.D. Thesis, Technische Hogeschool te Delft, 1955.
- [18] L. MALIGRANDA, *Orlicz Spaces and Interpolation*, Sem. Math. 5, Univ. of Campinas, Campinas SP, Brazil, 1989.

- [19] L. MALIGRANDA AND L. E. PERSSON, *Generalized duality of some Banach function spaces*, Proc. Konin Nederlands **92** (1989), 323–338.
- [20] L. E. PERSSON, *Some elementary inequalities in connection with X^p -spaces*, In: Constructive Theory of Functions, 1987, 367–376.
- [21] L. E. PERSSON, *On some generalized Orlicz classes and spaces*, Research Report 1988-3, Department of Mathematics, Luleå University of Technology, 1988.
- [22] M. M. RAO AND Z. D. REN, *Theory of Orlicz Spaces*, Marcel Dekker Inc., New York, Basel, Hong Kong, 1991.