

NEW CONVOLUTIONS AND NORM INEQUALITIES

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Abstract. We introduce new three types of convolutions for which – together with the classical convolution – we obtain new convolution inequalities. This is done within a framework from the theory of reproducing kernels which helps us to perform the mentioned inequalities in a very global way.

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