

## THE INVERSE THEOREM OF APPROXIMATION THEORY IN SMIRNOV–ORLICZ CLASSES

SADULLA Z. JAFAROV

*Abstract.* Let  $\Gamma$  be a Dini-smooth curve in the complex plane  $\mathbb{C}$ . In this study we prove inverse theorem of approximation theory by polynomials in Smirnov-Orlicz classes  $E_M(G)$ .

*Mathematics subject classification (2010):* 30E10, 41A10, 41A25, 46E30.

*Keywords and phrases:* Dini-smooth curve, Cauchy singular operator, Faber polynomials, inverse theorem, Orlicz space, Boyd indices, modulus of continuity.

### REFERENCES

- [1] S. JA. AL'PER, *Approximation in the Mean of Analytic functions of Class  $E_p$* , Gos. Izdat. Fiz.-Mat. Lit., Moscow, 1960, 272–286 (in Russian).
- [2] J. E. ANDERSSON, *On the degree of polynomial approximation in  $E^p(D)$* , J. Approx. Theory **19** (1977), 61–68.
- [3] M. I. ANDRASKO, *On the approximation in the mean of analytic functions in regions with smooth boundaries*, Problems in mathematical physics and function theory, Izdat. Akad. Nauk. Ukrain. RSR, Kiev, 1963, 1, p. 3 (in Russian).
- [4] R. AKGÜN AND D. M. ISRAFILOV, *Approximation and moduli of fractional orders in Smirnov-Orlicz classes*, Glas. Mat. **43**, 63 (2008), 121–136.
- [5] G. M. GOLUZIN, *Geometric Theory of Functions of a Complex Variable*, Translation of Mathematical Monographs vol. 26, R.I.: AMS, Providence, 1968.
- [6] I. I. IBRAGIMOV, D. I. MAMEDKHANOV, *Constructive characterization of a certain class of functions*, Dokl. Akad. Nauk **16** (1976), 820–823.
- [7] D. M. ISRAFILOV, *Approximation properties of generalized Faber series in an integral metric*, Izv. Akad. Nauk Azerb. Ser. Fiz.-Tekh. Mat. Nauk **2** (1987), 10–14 (in Russian).
- [8] D. M. ISRAFILOV, *Approximation by  $p$ -Faber polynomials in the weighted Smirnov class  $E^p(G, w)$  and the Bieberbach polynomials*, Constr. Approx. **17**, 3 (2001), 335–351.
- [9] D. M. ISRAFILOV, *Approximation by  $p$ -Faber Laurent rational functions in the weighted Lebesgue spaces*, Czechoslovak Math. J. **54**, 3(129) (2004), 751–765.
- [10] D. M. ISRAFILOV AND A. GUVEN, *Approximation in weighted Smirnov classes*, East J. Approx. **11**, 1 (2005), 91–102.
- [11] D. M. ISRAFILOV, B. OKTAY AND R. AKGÜN, *Approximation in Smirnov-Orlicz classes*, Glas. Mat. **40** (60) (2005), 87–102.
- [12] D. M. ISRAFILOV, ALI GUVEN, *Approximation by trigonometric polynomials in weighted Orlicz spaces*, Studia Math. **174**, 2 (2006), 147–168.
- [13] D. M. ISRAFILOV AND R. AKGÜN, *Approximation in weighted Smirnov-Orlicz classes*, J. Math. Kyoto Univ. **46**, 4 (2006), 775–770.
- [14] S. Z. JAFAROV, *Approximation by rational functions in Smirnov-Orlicz classes*, J. Math. Anal. Appl. **379** (2011), 870–877.
- [15] S. Z. JAFAROV, S. M. NIKOL'SKII, *Type inequality and estimation between the best approximation of a functions in norms of different spaces*, Math. Balkanica (N.S.) **21**, 1-2 (2007), 173–182.
- [16] S. Z. JAFAROV, *Approximations of Harmonic Functions Classes with Singularities on Quasiconformal Curves*, Taiwanese J. Math. **12**, 3 (2008), 829–840.

- [17] S. Z. JAFAROV, *Approximation by polynomials and rational functions in Orlicz spaces*, J. Comput. Anal. Appl. (JoCAAA) **13**, 5 (2011), 953–962.
- [18] M. A. KRASNOSELSKII AND YA. B. RUTICKII, *Convex Functions and Orlicz Spaces*, P. Noordhoff Ltd., Groningen, 1961.
- [19] V. KOKILASHVILI, *On analytic functions of Smirnov-Orlicz classes*, Studia Math. **31** (1968), 43–59.
- [20] A. YU. KARLOVICH, *Algebras of singular integral operators with piecewise continuous coefficients on reflexive Orlicz spaces*, Math. Nachr. **179** (1996), 187–222.
- [21] A. YU. KARLOVICH, *Algebras of singular integral operators with PC coefficients in rearrangement-invariant spaces with Muckenhoupt weights*, J. Operator Theory **47** (2002), 303–323.
- [22] W. MATUSZEWSKA AND W. ORLICZ, *On certain properties of  $\phi$ -functions*, Bull. Acad. Polon. Sci., Ser. Math. Aster. et Phys. **8-7** (1960), 439–443.
- [23] D. I. MAMEDKHANOV, *Approximation in complex plane and singular operators with a Cauchy kernel*, Dissertation Doct. Phys-math. nauk. The University of Tbilisi, 1984 (in Russian).
- [24] H. N. MHASKAR, *Introduction to the Theory of Weighted Polynomial Approximation*, Series in Approximation and Decompositions 7, World Sci., River Edge, NJ, 1996.
- [25] CH. POMMERENKE, *Boundary Behavior of Conformal Maps*, Berlin, Springer-Verlag, 1992.
- [26] A.-R. K. RAMAZANOV, *On approximation by polynomials and rational functions in Orlicz spaces*, Anal. Math. **10** (1984), 117–132.
- [27] M. M. RAO AND Z. D. REN, *Theory of Orlicz Spaces*, Marcel Dekker, New York, 1991.
- [28] P. K. SUETIN, *Series of Faber Polynomials*, Gordob and Breack, I. Reading, 1998.
- [29] S. E. WARSCHAWSKI, *Über das Ranverhalten der Ableitung der Abbildungsfunktion bei konformer Abbildung*, Math. Z. **35** (1932), 321–456.