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ON ω–QUASICONVEX FUNCTIONS

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Abstract. In the paper we introduce convexity-like notions based on modification of quasiconvexity.

DEFINITION. Let I be a real interval and $\omega \geqslant 0$ a given number. We say that a function $f : I \to \mathbb{R}$ is ω-*quasiconvex,* ω-*quasiconcave,* respectively, if

$$f(tx+(1-t)y) \leqslant \max(f(x), f(y)) - \omega \min(t, 1-t)|x-y|,$$
$$f(tx+(1-t)y) \geqslant \max(f(x), f(y)) - \omega \max(t, 1-t)|x-y|,$$
$$\text{for } x, y \in I, t \in (0,1).$$

If $f : I \to \mathbb{R}$ is simultaneously ω-quasiconvex and ω-quasiconcave then we say that f is ω-*quasiaffine*.

We characterize these notions, in particular we show that ω-quasiconcave functions coincide with Lipschitz functions with constant ω. We conclude the paper with the following separation type result.

THEOREM. Let $f : I \to \mathbb{R}$ be ω-quasiconvex function and $g : I \to \mathbb{R}$ ω-quasiconcave such that $f \geqslant g$.

Then there exists an ω-quasiaffine function $h : I \to \mathbb{R}$ such that $f \geqslant h \geqslant g$.

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