

THE CONVEXITY AND THE CONCAVITY DERIVED FROM NEWTON'S INEQUALITY

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Abstract. By Newton's inequality, a sequence $\{a_i\}_{i=0}^n$ of nonnegative real numbers is unimodal if its generating function $\sum_{i=0}^n a_i x^i$ has only real zeros. This paper is devoted to show that there exist two indices s and t with $s \leq t$, such that $a_0, a_1, \dots, a_{s-1}, a_s$ and a_t, a_{t+1}, \dots, a_n are convex, while $a_{s-1}, a_s, \dots, a_t, a_{t+1}$ is concave.

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