

OPERATOR INEQUALITIES FOR J -CONTRACTIONS

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Abstract. A selfadjoint involutive matrix J endows \mathbb{C}^n with an indefinite inner product $[\cdot, \cdot]$ given by $[x, y] = \langle Jx, y \rangle$, $x, y \in \mathbb{C}^n$. Characterizations of the J -chaotic order $\text{Log}(A) \geqslant^J \text{Log}(B)$ are presented for J -selfadjoint matrices A, B with positive eigenvalues, in terms of operator functions involving the α -power mean and the J -relative entropy. An indefinite complete form of the Furuta inequality and some exponential operator inequalities for J -selfadjoint matrices are also obtained. The parallelism between the inequalities in Hilbert spaces and the corresponding indefinite versions in Krein spaces is pointed out.

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