

EXTENSIONS OF THE HERMITE–HADAMARD INEQUALITY WITH APPLICATIONS

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Abstract. The main aim of this paper is to give improvements of various forms of the Hermite–Hadamard inequality, namely, that of Fejèr, Lupaş, Brenner–Alzer, Beesack–Pečarić. It is interesting that these improvements also imply the Hammer–Bullen inequality which deals with a comparison of the left-hand and the right-hand side of the Hermite–Hadamard inequality. These improvements are given in terms of positive linear functionals. Obtained results are used in constructing a new family of exponentially convex functions.

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