

A HARDY INEQUALITY FOR THE GRUSHIN TYPE OPERATORS

LI ZHU

Abstract. We prove a Hardy inequality related to Carnot-Carathéodory distance for the Grushin type operators like $\Delta_x + |x|^2 \partial_t^2$. The procedure is based on a representation formula for such operators.

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