SOME INEQUALITIES FOR THE
GENERALIZED DISTORTION FUNCTIONS

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Abstract. In this paper, we establish some functional inequalities for the generalized Hersch-Pfluger distortion function $\varphi_K(a,r)$, and prove a submultiplicative property for the generalized Agard distortion function $\eta_K(a,x)$.

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