Mathematical
Inequalities
& Applications
Volume 15, Number 4 (2012), 959–972
doi:10.7153/mia-15-82

AN INTEGRAL–TYPE OPERATOR FROM BLOCH SPACES TO $\mathcal{Q}_p$ SPACES IN THE UNIT BALL

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Abstract. Let $H(B)$ denote the space of all holomorphic functions on the unit ball B of $\mathbb{C}^n$. Let $\alpha > 0$, $f \in H(B)$ with homogeneous expansion $f = \sum_{k=0}^{\infty} f_k$. The fractional derivative $\mathcal{D}^\alpha f$ is defined as

$$\mathcal{D}^\alpha f(z) = \sum_{k=0}^{\infty} (k+1)^\alpha f_k(z).$$

Let φ be a holomorphic self-map of B and $g \in H(B)$ such that $g(0) = 0$. In this paper we consider the following integral-type operator

$$\mathcal{D}^\alpha_{\varphi,g} f(z) = \int_0^1 \mathcal{D}^\alpha f(\varphi(tz)) g(tz) \frac{dt}{t}, \quad f \in H(B).$$

The boundedness of the operator $\mathcal{D}^\alpha_{\varphi,g}$ from the Bloch space to the spaces $\mathcal{Q}_p$ and $\mathcal{Q}_{p,0}$ are investigated. In particular, the boundedness and compactness of the operator $\mathcal{D}^1_{\varphi,g}$ on the Bloch spaces are completely characterized.

Mathematics subject classification (2010): Primary 47B38, Secondary 30H30.
Keywords and phrases: Composition operator, integral-type operator, Bloch space, $\mathcal{Q}_p$ space.

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Paper MIA-15-82

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